ALL SAINTS' COLLEGE

Mathematics Specialist

Test 4 2019

Integration Techniques & Applications of Integral Calculus

NAME: _____ TEACHER: Mrs Da Cruz

Resource Free Section

26 marks
25 minutes

Question 1 **[3, 2, 6, 4, 5, 2, 4 = 26 marks]**

(a) $\int_{-\pi}^{\pi} |\sin x| dx$ (Hint: sketch the function first.) [3]

(b) Given that $\frac{d}{dx}[xe^x - e^x] = xe^x$, find $\int xe^x + x\,dx$. [2]

(c) Use the substitution $u = \ln x$ to determine $\int \frac{\sqrt{\ln x} + ln\sqrt{x}}{x} dx$. [6]

(c) $\int \sin^3 2t \, dt$ [4]

(d) Using partial fractions, find $\int \frac{1}{2x^2-x-6} \, dx$ [5]

(e) $\int \frac{e^{2x}}{3+2e^{2x}} dx$ [2]

(f) $\int_0^1 \frac{1-x}{x+1} dx$ $(Hint: You\ could\ use\ \ u = x + 1)$ [4]

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Test 4 2019

Integration Techniques & Applications of Integral Calculus

NAME: _____ TEACHER: Mrs Da Cruz

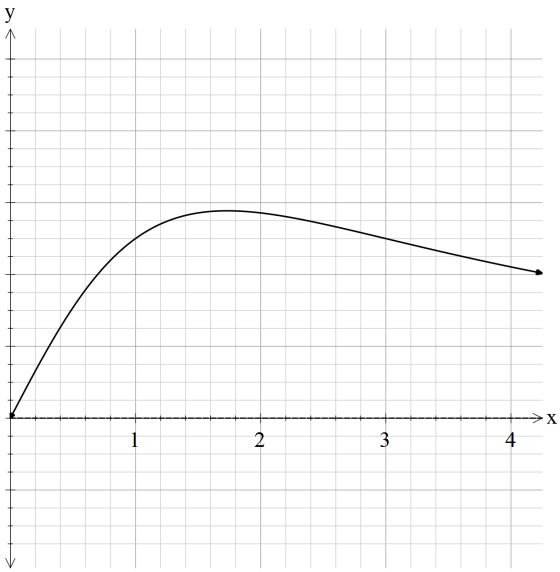
Resource Rich Section

19 marks
25 minutes

One unfolded A4 page of notes, SCSA formulae booklet and ClassPad calculator permitted.
Show sufficient working for marks to awarded.

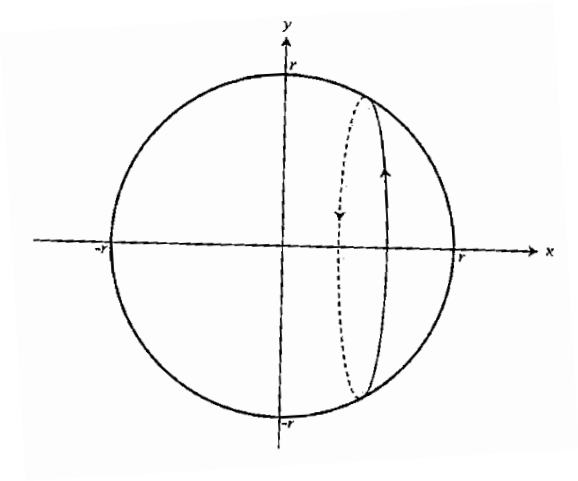
Question 2 **[2 marks]**

Consider the area under the curve $f(x) = \frac{2x}{3+x^2}$ between $x = 1$ and $x = 3$. Using four mid-point rectangles **approximate the area.**



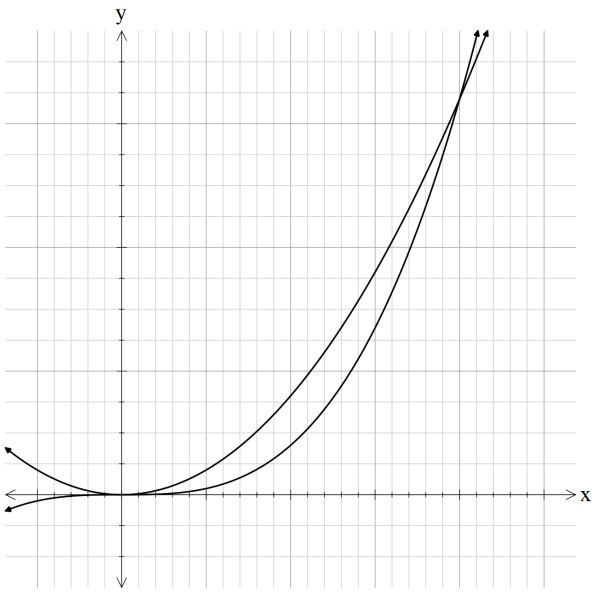
Question 3 **[4 marks]**

Show that the volume of a sphere, $V = \frac{4\pi r^3}{3}$, may be generated by rotating the circle $x^2 + y^2 = r^2$ about the x-axis.



Question 4 **[5 marks]**

Consider the region bounded by the curves $f(x) = 4x^2$ and $g(x) = x^3$ for $x \geq 0$. Determine the volume of the solid formed when the region is rotated about the y-axis.



Question 5 **[8 marks]**

Find the area of the lens shape formed between two circles, $x^2 + y^2 = 1$ and $x^2 + (y-1)^2 = 1$.

(Hints: You will need to first find the relevant semi-circle equations. Use the substitution $x = \sin\theta$)